Lecture Two

Geometry of the Heavens

Scope: The simplest explanation for the motions of the heavens is the geocentric one: The Sun, Moon, stars, and planets are orbiting around Earth at the center. The scientific challenge is to develop a physical model that can explain and predict these motions, especially the retrograde motions of the planets. Over 500 years of studies, philosophies, and calculations underlie the publishing of Ptolemy's *Almagest* around 150 C.E. The book systematically develops a mathematical explanation based upon the geometry of circles, off-center circles, and circles upon circles. This Earth-centered model provided predictions of sufficient accuracy that it remained the standard for another 1500 years. More generally, the universe was shown to be comprehensible, and heavenly motions could be made transparent by humankind's ingenuity.

Outline

I. The simplest model of the solar system is an Earth-centered one.
- **A.** The Sun, Moon, stars, and planets appear to orbit around Earth.
- **B.** A scientific model of the solar system must be able to account for observed motions and predict future positions.
- **C.** The geocentric solar system devised by the ancients serves as an excellent example of building up a scientific model.
- **D.** It is a myth that the geocentric model became too complicated and collapsed under the weight of tweaks and inaccuracies.

II. Many astronomical observations can be made by careful observation and judicious application of geometry.
- **A.** The shape of Earth's shadow during a lunar eclipse argues that Earth must be spherical.
- **B.** The relative distances to the Moon and Sun can be calculated by triangulation.
- **C.** A total solar eclipse shows that the ratio of sizes of the Moon and Sun is the same as the ratio of their distances.
- **D.** The size of the Moon relative to Earth can be determined from careful observations of lunar eclipses.
- **E.** The size of Earth can be calculated using noontime shadows.

III. The geocentric model in Ptolemy's *Almagest* (150 C.E.) brought together 500 years of astronomical knowledge and is based on the geometry of Euclid's *Elements* (300 B.C.E.).
- **A.** The geocentric model of the ancients considers the objects visible with the unaided human eye: the Sun, the Moon, Mercury, Venus, Mars, Jupiter, Saturn, and the stars.
  1. Each object moves across the sky at a slightly different speed.
  2. The ordering of object distances was based on the idea that the closest objects showed the fastest motions.
  3. The observed speed of motion can vary considerably over time.
- **B.** The eccentric circle can explain the motion of the Sun.
  1. If the Sun showed circular motion, the seasons would all be the same length.
  2. The observed lengths of the seasons differ by several days.
  3. The Sun moves faster, relative to the stars, in January than it does in July.
  4. An eccentric circle uses circular motions in which the observer is located away from the center of the circle.
  5. For an eccentric circle, the motion across the sky is fastest when closest, and slowest when farthest.
- **C.** The epicycle and deferent system can explain the basic motion of the Moon.
  1. Our modern idea of a moon orbiting a planet that is orbiting a star is a natural expression of the epicycle and deferent system.
  2. The deferent circle is the main orbit, and the epicycle circle is the suborbit, whose center moves along the deferent circle.
  3. The moon's motion can be reasonably fit by one epicycle but even better fit by adding a second epicycle to the system.
  4. Using epicycle and deferent, one can also exactly reproduce the motion of an eccentric circle.
- **D.** The equant is like an extension of the idea of an eccentric circle.
  1. The equant uses circular motion in which both the observer and the center of the rotation are displaced equally from the center of the orbit circle.
  2. The equant produces larger variation between the fastest and slowest motion of an object across the sky.

E. The planets were modeled with an equant and epicycle-deferent.

1. Mercury and Venus are always observed near the Sun, and hence their motion along the deferent was tied to the motion of the Sun.
2. When Mars, Jupiter, and Saturn are opposite the Sun on the sky, they show short periods of retrograde motion, moving backward across the sky relative to the stars.
3. Retrograde motion is produced in the geocentric model when the angular speed along the epicycle is significantly faster than the angular speed along the deferent.
4. To tie retrograde motion to the time of opposition, the motions of Mars, Jupiter, and Saturn along their epicycles were tied to the motion of the Sun.

IV. Though eventually shown to be incorrect, the geocentric model of the solar system was an intellectual triumph.

A. A full scientific model of the solar system was based on three geometrical constructs: eccentric circle, epicycle-deferent, and equant.

B. Ptolemy's work was entitled *Mathematical Systematic Treatise*, but it was later called *Almagest* by Arabic astronomers.

C. The geocentric model provided accurate enough predictions that it remained the standard for 1500 years.

D. The geocentric model showed the basic tenet underlying astronomy: that the universe is understandable, explainable, and predictable.

Suggested Readings:

Bennett, Donahue, Schneider, and Voit, *The Cosmic Perspective*, chap. 3.

Crowe, *Theories of the World.*

Hakim, *The Story of Science.*

Ptolemy, *Almagest.*

Weintraub, *Is Pluto a Planet?* chap. 2.

Wilson, *Astronomy through the Ages*, chap. 3.

Questions to Consider:

1. Before this lecture, what preconceptions about the Earth-centered model did you have? Did any of those ideas turn out to be misconceptions?
2. The Ptolemaic model can be considered simple, as it relies on three geometrical constructs, or complex, as the motions produce intricate patterns. Which position would you support?
3. Is accurate prediction of events enough for a scientific model, or does it have to truthfully represent reality?