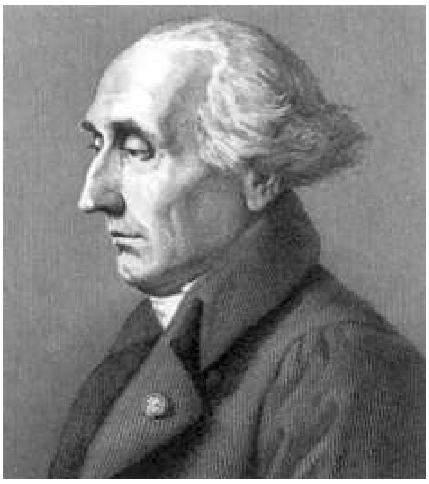
THE LAGRANGE POINTS

Joseph-Louis Lagrange made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics. In 1766, on the recommendation of **Leonhard Euler** and **Jean-Baptiste le Rond d'Alembert**, Lagrange succeeded Euler as the director of mathematics at the Prussian Academy of Sciences in Berlin, Prussia, where he stayed for over twenty years, producing volumes of work and winning several prizes of the French Academy of Sciences.



Joseph-Louis Lagrange (1736 – 1813), was an Italian / French Enlightenment Era mathematician and astronomer

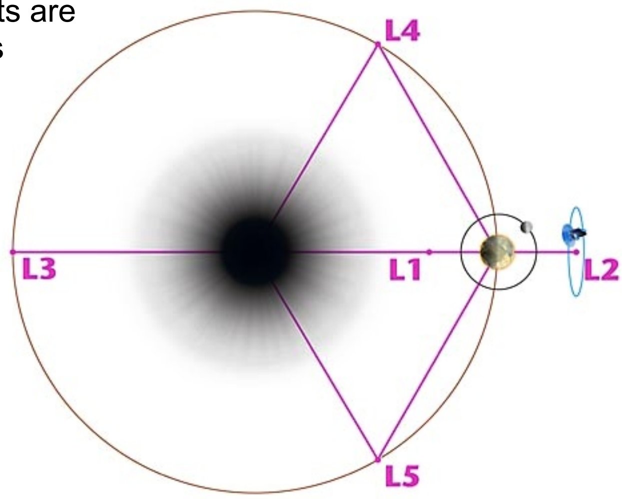
Lagrange's treatise on analytical mechanics, written in Berlin and first published in 1788, offered the most comprehensive treatment of classical mechanics since **Isaac Newton** and formed a basis for the development of mathematical physics in the nineteenth century.

In 1787, at age 51, he moved from Berlin to Paris and became a member of the French Academy of Sciences. He remained in France until the end of his life. He was significantly involved in the decimalisation in Revolutionary France, became the first professor of analysis at the École Polytechnique upon its opening in 1794, was a founding member of the Bureau des Longitudes, and became Senator in 1799.

SCIENTIFIC CONTRIBUTION

Lagrange was one of the creators of the calculus of variations, deriving the Euler–Lagrange equations for extrema of functionals. He also extended the method to take into account possible constraints, arriving at the method of Lagrange multipliers. Lagrange invented the method of solving differential equations known as variation of parameters, applied differential calculus to the theory of probabilities and attained notable work on the solution of equations. He proved that every natural number is a sum of four squares. **He studied the three-body problem for the Earth, Sun and Moon (1764) and the movement of Jupiter's satellites (1766), and in 1772 found the special-case solutions to this problem that yield what are now known as Lagrangian points**.

Lagrange points are five special points where a small mass can orbit in a constant pattern with two larger masses. These points are positions where the gravitational pull of two large masses precisely equals the centripetal force required for a small object to move with them. Of the five Lagrange points, three are unstable and two are stable. The unstable Lagrange points - L1, L2, L3 - lie along the line connecting the two large masses. The stable Lagrange points - L4 and L5 - form the apex of two equilateral triangles that have the large masses at their vertices. L4 leads the orbit of earth and L5 follows.

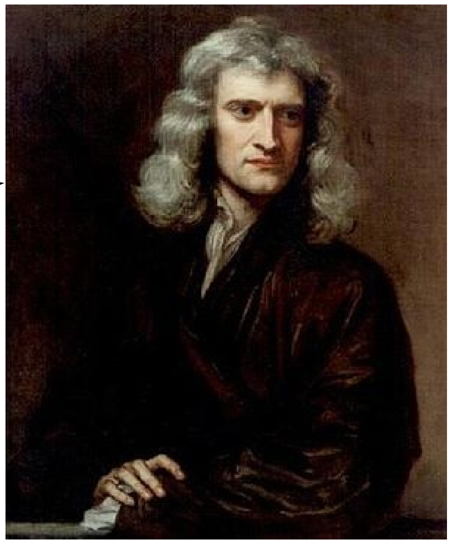


Lagrange Points of the Earth-Sun system (not to scale!).

The L1 point affords an uninterrupted view of the Sun and is currently home to the Solar and Heliospheric Observatory Satellite SOHO. **The L2 point of the Earth-Sun system was the home to the WMAP spacecraft, is the current home of Planck, and future home of the James Webb Space Telescope. L2 is ideal for astronomy because a spacecraft is close enough to readily communicate with Earth, can keep Sun, Earth and Moon behind the spacecraft for solar power and (with appropriate shielding) provides a clear view of deep space for the telescopes**. The L1 and L2 points are unstable on a time scale of approximately 23 days, which requires satellites orbiting these positions to undergo regular course and attitude corrections.

NASA is unlikely to find any use for the L3 point since it remains hidden behind the Sun at all times. The L4 and L5 points are home to stable orbits so long as the mass ratio between the two large masses remains stable. Objects found orbiting at the L4 and L5 points are often called Trojans after the three large asteroids Agamemnon, Achilles and Hector that orbit in the Jupiter-Sun system. There are hundreds of Trojan Asteroids in the solar system, most with Jupiter and Mars. In 2010 the WISE telescope confirmed a Trojan asteroid (2010 TK7) around Earth's leading Lagrange point L4.

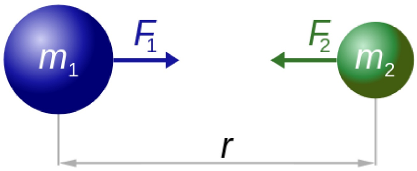
There are two more points in the Sun / Earth system, where the gravitational forces cancel each other out, at the centre of the Sun and the Earth. These are not mentioned in the above treatment as they probably are not considered of any astronomical significance. But they are very significant if we want to talk about the nature of Gravity as one of the four natural forces.



In 1687, English mathematician **Sir Isaac Newton** published *Principia*, which hypothesizes the inverse-square law of universal gravitation. In his own words:

> *I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centres about which they revolve: and thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the Earth; and found them answer pretty nearly.*

The equation is the following: Where F is the force, m1 and m2 are the masses of the objects interacting, r is the distance between the centres of the masses and G is the gravitational constant: 6.67408 $\times 10^{-11}$ $m^3 \cdot kg^{-1} \cdot s^{-2}$ (metres cubed per kilogram per second squared)



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

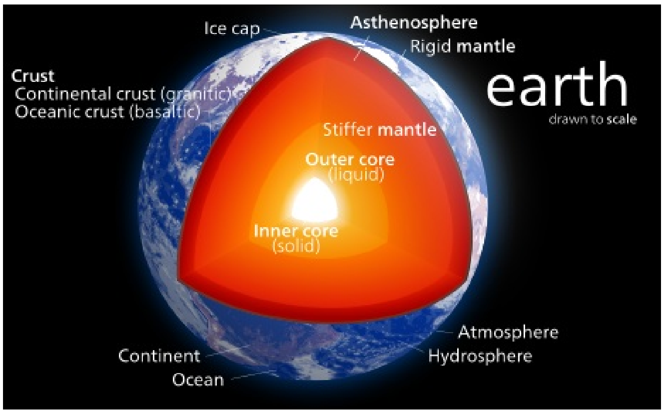
The gravitational constant G is a key quantity in Newton's law of universal gravitation

Newton's theory enjoyed its greatest success when it was used to predict the existence of planet Neptune, based on motions of Uranus that could not be accounted for by the actions of the other planets. **John Couch Adams** and **Urbain Le Verrier** predicted the general position of the planet, that led **Johann Gottfried Galle** to the discovery of Neptune.

By the end of the 19th century a small discrepancy in Mercury's orbit pointed out flaws in Newton's theory. The issue was resolved in 1915 by **Albert Einstein's** new theory of General Relativity, accounting for the discrepancy.

The gravitational constant appears in Newton's law of universal gravitation, but it was not measured until seventy-one years after Newton's death by **Henry Cavendish**, using a torsion balance invented by the geologist **Rev. John Michell**. Cavendish's aim was not actually to measure the gravitational constant, but rather to measure Earth's density relative to water, through the precise knowledge of the gravitational interaction.

Although Newton's theory has been superseded by Einstein's General Relativity, most modern non-relativistic gravitational calculations are still made using Newton's theory. His Shell Theorem has a particular application to astronomy: **If the body is a spherically symmetric shell (a hollow ball), no net gravitational force is exerted by the shell on any object inside, regardless of the object's location within the shell.** A corollary of this is that inside a solid sphere of constant density, the gravitational force varies linearly with distance from the centre, becoming zero at the centre of mass. **An example of this reasoning is: if you stood on a set of bathroom scales in an elevator going down to the centre of the Earth, your body weight (as shown on the scales) would be less and less the deeper you get and drop to zero at the very centre of the Earth!**



Why then is the pressure towards the centre increasing, when gravitational attraction actually decreases? Should there then not also be zero pressure at the Centre? Hollow Earth Mythology? Note: the rising temperature within the Earth are caused by radioactive decay of elements, not by the pressure! Perhaps the binding energy of the Strong Force - which is supposed to make up over 50% of the mass of an elementary particle - comes onto it?

Scientific understanding of the internal structure of the Earth is based on observations of topography, samples brought to the surface by volcanic activity, analysis of the seismic waves that pass through the Earth and measurements of the gravitational and magnetic fields of the Earth.

AK, with Wikipedia Notes