

KEPLER'S LAWS OF PLANETARY MOTION

In astronomy, Kepler's laws of planetary motion are three scientific laws describing the motion of planets around the Sun. Kepler's laws are now traditionally enumerated in this way:

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period is proportional to the cube of the semi-major axis of its orbit

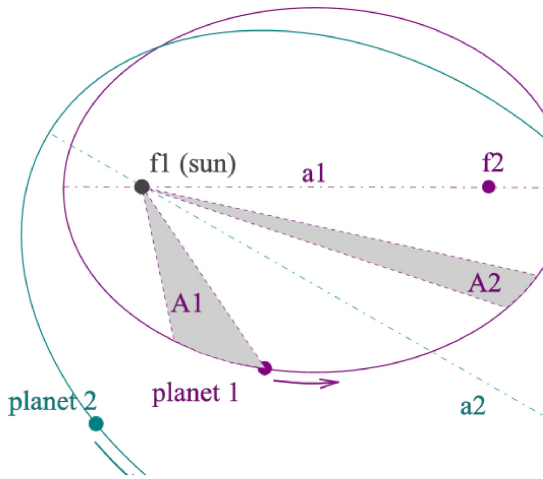


Figure 1: Illustration of Kepler's three laws with two planetary orbits.

(1) The orbits are ellipses, with focal points f_1 and f_2 for the first planet. The Sun is placed in focal point f_1 .

(2) The two shaded sectors A_1 and A_2 have the same surface area and the time for planet 1 to cover segment A_1 is equal to the time to cover segment A_2 .

(3) The total orbit times for planet 1 and planet 2 have a ratio $a_1^{-3/2} : a_2^{-3/2}$.

Table of Solar Planet Orbits and Speeds

Planet	Radius of Orbit Relative to that of Earth's	Length of Year Relative to Earth's Year	Orbital Velocity Relative to That of Earth's
Mercury	0.387	0.2409	1.607
Venus	0.723	0.616	1.174
Earth	1.0	1.0	1.000
Mars	1.524	1.9	0.802
Jupiter	5.203	12.0	0.434
Saturn	9.539	29.5	0.323
Uranus	19.18	84	0.228
Neptune	30.06	165	0.182
Pluto	39.52	248	0.159

Thus Mercury's orbital speed is $1.607(67,000) = 107.7$ thousand miles per hour, as befits a planet named for the god of speed. Mars is a bit of a laggard. Its speed is only $0.802(67,000) = 53.7$ thousand miles per hour. Pluto is veritably creeping around its orbit at only 10.7 thousand miles per hour.

There is an interesting computation which can be performed using the above figures. Let us look at the products of the square roots of the relative radii and the orbital velocities. The computations are given below:

Thus the relative orbital speed is given by: $V = 1/R^{1/2}$

This is just Kepler's Law in a different form. Kepler's Law is that for each planet the square of the length of its year is equal to the cube of the radius of its orbit. From Kepler's Law if you know how far a planet is from the sun you can tell how long it would take for that planet to go around the sun.

Thus in relative terms $T^2 = R^3$ so $T = R^{3/2}$ and hence $V = R/T = R/R^{3/2} = 1.0/R^{1/2}$; and for a planet twice as far from the sun as Earth the orbital velocity relative to that of Earth's is $1/\sqrt{2} = 0.707$.

Standing on Earth's surface at the equator (for comparison -- not an orbit) 465.1 m/s (1,040 mph) 1 day (24h)

Low Earth orbit 8,400 km circular orbit: 7.8 km/s (17,450 mph) 2 h 8 min

Geostationary 42,000 km 6,935 mph 23 h 56 min

Orbit of the Moon 399,000 km 2416 mph = 27.3 days